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STUDENT REPORT

DIFFERENTIAL EQUATIONS COURSE MODULE
FOR THE SUPERIOR STUDENT--
CURRICULUM DEVELOPMENT

MAJOR HARWOOD A. HEGNA 87-1150

"insights into tomorrow"

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The need to provide a more stimulating academic program for the superior cadets at the Air Force Academy has been identified. The report describes the concept of inserting a module of lessons into a course to transform it into a more challenging version. Three specific approaches for providing more challenge to students are discussed. The new curriculum course in differential equations for technical majors, Math 245, serves as the specific example. The modular concept can be applied to a broad range of courses.

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PREFACE

The author has a keen interest in seeking ways of academically challenging students enrolled in engineering, science, and mathematics college level programs. This need became evident while he served as a faculty member at the United States Air Force Academy. The present report describes one method of creating a more stimulating version of a course for the superior student. The new curriculum course in differential equations for technical majors, Math 245, serves as the specific example. The materials in Appendix A are planned for use in an advanced version of the course offered this fall. The author wishes to thank both Colonel Daniel Litwhiler, Head of the Department of Mathematical Sciences, for his support and Major Bob Cass, faculty member in the Department of Mathematical Sciences, for his technical and administrative assistance. The author also appreciates the typing support of Mr Phil Pancake, technician in the Department of Mathematical Sciences, who typed the course materials in Appendix A.

ABOUT THE AUTHOR

Harwood Allan Hegna [redacted]. He graduated from Washburn High School in Minneapolis in 1965 and attended the University of Minnesota from which he was conferred the degree of Bachelor of Aeronautical Engineering with High Distinction in June 1969. He continued his education in the Graduate School of the University of Minnesota and received a Master of Science in Aeronautical Engineering degree in December 1971 with a thesis article entitled "The Rarefied Gas Flow Through a Slit" published in the Journal of the Physics of Fluids. He was commissioned in the USAF through the ROTC program in June 1972 and entered active duty in August 1972. He served as an engineering plans and programs manager for the Ogden Air Logistics Center, Hill AFB, Utah. While on active duty, he completed the requirements and was awarded the degree of Master of Science in Mechanical Engineering from the University of Minnesota in March 1973. He attended Squadron Officer School in residence prior to his entering the School of Engineering, Air Force Institute of Technology (AFIT) in September 1976. He was married to Helen Ruth [redacted] of Orlando, Florida, on 25 August 1979. He graduated in June 1981 from AFIT with a PhD in Aerospace Engineering. While assigned to Wright Aeronautical Labs, Wright-Patterson AFB, Ohio, he conducted research in numerical methods for aerodynamics and published articles, titled "Numerical Solution of Incompressible Turbulent Flow Over Airfoils Near Stall" and "Numerical Prediction of Dynamic Forces on Arbitrarily Pitched Airfoils", in the American Institute of Aeronautics and Astronautics Journal. He served on the Air Force Academy faculty from May 1982 to July 1986 and received the rank of Associate Professor in July 1984. He taught courses in applied mathematics and a course in computational methods for aerodynamics. He was the academic Advisor in Charge for the Basic Sciences Divisional Major and directed the research effort within the Mathematics Department. Major Hegna possesses a diversity of experience in Air Force engineering.

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INSTRUCTIONAL SYSTEM DEVELOPMENT DOCUMENTATION

INTRODUCTION

The faculty of the United States Air Force Academy (USAFA) performed a thorough review of both the Core Curriculum and the individual major programs during the 1985-1986 academic year. The Dean of Faculty requested the review for the purpose of reducing the number of core courses required by all cadets for graduation. The faculty recommended the elimination of some courses and the development of others. The new Core Curriculum has seven fewer courses (7:22). The introductory differential equations (DE) course offered by the Department of Mathematical Sciences was dropped as a core course. The loss of previously offered courses and the availability of an average six new course offerings led to a review of each major program by separate curriculum committees. Each committee examined numerous issues ranging from overall curriculum goals and sequences of their major courses to the differential equations topics required to support their major. The engineering and basic sciences committees considered differential equations because the introductory DE core course was eliminated and they planned a more in-depth program of study in their majors. A more thorough course in differential equations for the engineering and science majors was mandated early by the Engineering Division and Basic Sciences Division Department Heads (13:--).

During the author's assignment on the faculty, he talked with several cadets who expressed concern with the lack of opportunity for a more in-depth study. They referred to an academic program with a perceived emphasis on numbers of courses and remedial courses for academically weak cadets. Some of these academically gifted cadets subsequently voluntarily left the Academy. The author has heard the Dean of Faculty on several occasions express his desire to find ways of better challenging these superior cadets. The Head of the Department of Mathematical Sciences has also spoken of this need.

Are there ways of building academic challenge for the superior student into the basic version of a course? The goal of this project has been to develop an affirmative answer. The chosen approach uses the Instructional System Development (ISD) model (8:1-5) to first build a set of educational objectives for the new basic course in differential equations. A module of lessons designed to encourage further study is then developed by continuing the ISD process. The module can be plugged into the basic version to create the desired advanced version with a relatively small administrative overhead cost. The following sections of this report discuss how the five steps of the ISD model have been applied to obtain the advanced course module.

STEP 1: ANALYZE SYSTEM REQUIREMENTS

In Step 1 the defined system is a first course in differential equations for technical majors. The need for the course itself is examined followed by a study of what topics do students need to study in such a course. The revised curriculum for each major offered at USAFA was examined to determine where differential equations first appeared explicitly as a prerequisite for a course in the major. The list of programs, together with the first course, is the following: aeronautical engineering (Aero 342, Aerodynamics); astronautical engineering, civil engineering, and engineering mechanics (Engr Mech 320, Dynamics); chemistry and biochemistry (Chem 335, Physical Chemistry); electrical engineering (El Engr 340, Circuits); mathematics (Math 346, Engineering Mathematics); and physics (Phys 357, Classical Mechanics). Clearly, the engineering and science majors require differential equations. The individual courses were then studied to identify the DE topics employed. Each course uses properties of first and higher order equations. In addition, the electrical engineering first course requires the Laplace transform method, Chemistry 335 uses separation of variables, and Engineering Mechanics 320 calls for vector and matrix forms. Many undergraduate technical programs build from a similar base of differential equation concepts although early use of individual techniques varies with the discipline.

An institutional requirement exists for a block of lessons on linear algebra. The Engineering and Basic Sciences Divisions reaffirmed this requirement during the curriculum review (13:--). The mathematics and operations research programs, however, do require a full theoretical algebra course and computational algebra course, respectively. For several years this small block has resided in the DE core course. Thus, the DE course for technical

majors has become the heir apparent for the linear algebra block. Linear algebra topics frequently are covered in DE texts, as the book analysis of topics will now reveal.

Several undergraduate differential equations textbooks were analyzed to discover what topics authors chose to include in a first course. The textbook sample varied from classics over 15 years old to a new book published in 1985. In every case the edition of the text was printed in 1982 or later. The topics presented by each author are listed under categories in Table 1. Each topic covered in four of the five books was included in the proposed course list. The topics indicated by an asterisk (*) were added for various reasons. The first-order variation of parameters method provides consistency with the other included uses of the method. Each textbook provides at least limited coverage of polynomial operators. The linear algebra constraint caused the topics to be added, as already discussed. Authors of DE textbooks frequently include linear algebra, as shown by the study, because it complements the subject of linear differential equations.

Topic	Source Number					Included
	1	2	3	4	5	
First-order DE						
Introduction	x	x	x	x	x	x
Separation of Variables	x	x	x	x	x	x
Variation of Parameters	x	x	x			*
Integrating Factors	x	x	x	x	x	x
Direction Fields	x	x	x	x		x
Exact Differentials	x		x	x	x	x
Existence/Uniqueness	x	x	x	x	x	x
Nth-order Linear DE						
Spring Models	x	x	x	x	x	x
Homogeneous Properties	x	x	x	x	x	x
Linear Independence of Functions	x	x	x	x	x	x
Homogeneous Constant Coefficients	x	x	x	x	x	x
Undetermined Coefficients	x	x	x	x	x	x
Variation of Parameters	x	x	x	x	x	x
Reduction of Order	x		x	x	x	x
Polynomial Operators			x		x	*
Circuit Models	x	x		x	x	x
Special Forms		x				

Table 1. Topics Covered in Textbooks

Topic	Source Number 1	2	3	4	5	Included
Linear Systems of DE						
Properties	x	x	x	x	x	x
Linear Independence of Vectors	x	x	x	x	x	x
Eigenvalue Method	x	x	x	x	x	x
Matrix exp(tA)					x	
Variation of Parameters	x	x	x	x	x	x
Inverse Method	x					
Laplace Transform (DE)	x	x	x	x	x	x
Laplace (System of DE)		x	x			
Power Series (Ordinary)	x	x	x	x	x	x
Power Series (Singular)	x	x	x	x	x	x
Numerical Methods						
Euler	x	x	x	x	x	x
Modified Euler	x	x	x	x	x	x
Runge-Kutta	x		x	x	x	x
Multi-step	x			x	x	
Truncation Error	x		x	x		
System of DE	x	x	x	x		x
Introduction to Partial DE	x	x	x	x		x
Fourier Series	x	x	x	x		x
Qualitative Studies	x	x		x		
Sturm-Liouville	x			x		
Linear Algebra						
Matrix Forms	x		x	x	x	x
Gauss Elimination	x			x	x	*
Gauss-Jordan			x	x	x	*
Determinants			x	x	x	*
Cramer's Rule			x		x	*
Matrix Inverse				x	x	*
System Properties			x	x	x	*
Vector Spaces					x	*
Subspaces					x	*
Bases					x	*
Dimension					x	*
Transformations					x	

* Secondary Included List

Sources: 1 Boyce (2:--), 2 Brauer (3:--), 3 Guterman (4:--), 4 Powers (5:--), and 5 Rabenstein (6:--)

Table 1. Topics Covered in Textbooks (Continued)

The need for an undergraduate differential equations course for technical majors has been demonstrated. A textbook study produced a large collection of possible topics for study in the proposed course (topics checked in "Included" column of Table 1). These system requirements produce the educational requirements during the second ISD step.

STEP 2: DETERMINE EDUCATIONAL REQUIREMENTS

Educational requirements come from the system requirements and provide the foundation for the actual course. In this step the course developer examines the priorities of the topics and the prior knowledge of the students.

The importance of topics can be determined from their early appearance in the academic programs and through discussions with department curriculum committees. The previous study of introductory courses in each technical major revealed a higher priority placed on first and second order linear differential equations, matrix algebra, and Laplace transforms. Also, the author attended several curriculum discussions during the fall of 1985 with representatives from the engineering and science departments. The representatives indicated that power series methods, approaches to linear DE systems, and an introduction to partial differential equations with Fourier series could come later in the curriculum (13:--). Furthermore, they requested instruction in related applied mathematics topics such as vector fields and Fourier transforms. The committee concluded that a following engineering mathematics course should also be studied. This early decision made the present task of prioritizing DE topics much more straightforward.

The priority of individual topics within a major category was also examined. The prime factor was the finite time of 42 hours in a one semester course. Within the first-order DE category, the direction fields method was eliminated because the pictorial representation is similar to plotted numerical results. Exact differentials is a rather specialized approach related to partial differentiation. Finally, the more theoretical existence and uniqueness topics are more easily studied, in the author's judgment, after the student has worked with solutions to many physical problems. In fact, a follow-up more theoretical course in DE for senior math majors already exists, Math 468, Intermediate Differential Equations. Consequently, both the exact differentials and existence/uniqueness categories were eliminated. Circuit applications in the higher order DE

category have been deleted in favor of the more easily visualized spring and mass problem. The reduction of order technique is related to the retained variation of parameters method. The reduced list of educational requirements appears in Table 2.

Topics	Topics
First-order DE	Numerical Methods
Introduction	Euler
Separation of Variables	Modified Euler
Variation of Parameters	Runge-Kutta
Integrating Factors	
	Linear Algebra
Nth-order Linear DE	General Properties
Homogeneous Properties	Matrix Methods
Polynomial Operators	Gauss Elimination
Linear Independence of Functions	Gauss-Jordan
Homogeneous Constant Coefficients	Determinants
Undetermined Coefficients	Cramer's Rule
Variation of Parameters	Matrix Inverse
Spring Models	Vector Spaces
	Subspaces
Laplace Transforms	Bases
	Dimension

Table 2. Educational Requirements

Before the list of requirements can be considered final, the prior knowledge of the students must be assessed. This assessment both confirms the assumed prerequisite knowledge of the students and also eliminates any topics that have already been learned. Each summer, personnel from the Mathematics Department administer a series of placement examinations to the new cadets during basic training. Every cadet takes the algebra and trigonometry exams. In addition, they may elect to take the Calculus I (Differential Calculus), Calculus II (Integral Calculus), and Calculus III (Multivariate Calculus) exams. The entry level math course for a student is based on these test scores along with transcripts of prior college work. A cadet must pass the algebra and trigonometry exams or complete the remedial course, Math 130, in order to enter Calculus I. The revised Core Curriculum includes Math 141, Calculus I, and Math 142, Calculus II. Thus, all potential students for the DE course

must validate or pass Math 142 eventually. Furthermore, the topics in Table 2 require no higher mathematics beyond differential and integral calculus of one variable. Thus, Math 142 has become the minimum prerequisite level of knowledge which the students should possess. This prerequisite choice also provides scheduling flexibility in many technical majors that require Calculus III. None of the topics in Table 2 are covered in the differential and integral calculus courses. Approximately one half of the entering cadets have seen some of the algebra topics according to classroom surveys by the author. Specifically, simple elimination, determinants, and some matrix algebra have been previously studied. These subjects are integrated into this linear algebra block and will serve as a review for some students.

The first two steps of the ISD process have shown the differential equations course for technical majors is needed and should be built from the educational requirements in Table 2.

STEP 3: FORMULATE OBJECTIVES AND TESTS

Specific educational objectives are formulated in Step 3 of the ISD process from the educational requirements developed in Step 2. The objectives are written in terms of Bloom's cognitive levels of learning (1:--) with associated samples of behavior. Most actual solution methods have objectives written at the application level of learning. At this level the student must identify a suitable technique and use it correctly to solve a given problem. Application level understanding requires the student to "put theory into practice" (10:9-1) and provides the foundation for further studies in engineering and the sciences. Most concepts, such as "specific solution", have objectives formulated at the comprehension level of learning. Here a student conveys meaning to an identified concept.

The objectives are set forth below, along with a sample of behavior (SOB), following the order of the educational requirements given in Table 2.

Basic Course Objectives

First-order Differential Equations.

1. Comprehend the concept of the general solution of an ordinary differential equation.
    SOB: Show that a given function leads to an equality

when substituted into the differential equation and has the same number of arbitrary constants as the order of the differential equation.

2. Comprehend the concept of the specific solution of an ordinary differential equation.
SOB: Show that a given function leads to an equality when substituted into the differential equation and satisfies the set of initial conditions.

3. Comprehend how separating the variables may solve a first-order differential equation.
SOB: Solve a first-order differential equation using separation of variables when directed to use that method.

4. Comprehend the concept of a linear differential equation.
SOB: Show where a given differential equation deviates from the linear form.

5. Apply the variation of parameters method to solve a first-order linear differential equation.
SOB: Solve a differential equation using the variation of parameters method by first identifying the equation as a first-order linear problem.

6. Apply the integrating factor method to solve a first-order linear differential equation.
SOB: Solve a differential equation using the integrating factor method by first identifying the equation as a first-order linear problem.

Nth-order Linear Differential Equations.

7. Comprehend how superposition and proportion build the general solution for a homogeneous linear differential equation.
SOB: Create the general solution by forming an arbitrary linear combination of n linearly independent solutions to an nth-order homogeneous linear differential equation.

8. Comprehend the concept of the linear independence of functions.
SOB: Show that any one function in a given set cannot be written as a linear combination of the remaining functions.

9. Apply the characteristic polynomial method to obtain the general solution for a homogeneous linear differential equation with constant coefficients.
SOB: Solve an nth-order factored polynomial operator form of a homogeneous linear differential equation using the characteristic polynomial method.

10. Apply the undetermined coefficients method to obtain a particular solution for a nonhomogeneous linear differential equation with constant coefficients.

SOB: Build an undetermined coefficients form for the particular solution and deduce the values of the coefficients for a suitable differential equation.

11. Apply the variation of parameters method to solve a linear differential equation.

SOB: Solve a linear differential equation using the variation of parameters method.

12. Comprehend that the damped, spring and mass system illustrates a second-order linear differential equation problem.

SOB: Solve a specific spring and mass system problem using a suggested method suitable for linear differential equations.

Laplace Transforms.

13. Apply the Laplace transform method to solve a linear differential equation with constant coefficients and initial conditions.

SOB: Solve an initial value problem using Laplace transforms by first identifying the problem as a linear constant coefficient differential equation.

Numerical Methods.

14. Comprehend how Euler and modified Euler numerical methods approximate a specific solution to a first-order differential equation.

SOB: Complete one step of the specified Euler method using the initial condition to obtain an approximate solution.

SOB: Complete one step of the specified modified Euler method using the initial condition to obtain an approximate solution.

15. Comprehend how the Runge-Kutta numerical method approximates a specific solution to a first-order differential equation.

SOB: Complete one step of the specified Runge-Kutta method using the initial condition to obtain an approximate solution.

Linear Algebra.

16. Comprehend the concept of equivalent linear systems.

SOB: Show that one given system can be obtained from a second system using elementary operations.

17. Comprehend that any linear system will have either no solution, one solution, or infinitely many solutions.
SOB: Explain that a homogeneous linear system with one given nontrivial solution has infinitely many solutions.

18. Comprehend the concept of matrix algebraic operations.
SOB: Add together two matrices of equal dimension.
SOB: Compute the product of two suitably dimensioned matrices.

19. Apply the Gauss elimination method to solve a linear algebraic system of equations.
SOB: Solve a linear algebraic system of equations using the Gauss elimination method.

20. Apply the Gauss-Jordan reduction method to solve a linear algebraic system of equations.
SOB: Solve a linear algebraic system of equations using the Gauss-Jordan reduction method.

21. Comprehend how the expansion method is used to calculate the value of a determinant.
SOB: Compute the value of the determinant for a 4x4 matrix.

22. Apply Cramer's rule to solve a nonsingular linear algebraic system of equations.
SOB: Solve a linear algebraic system of equations using Cramer's rule by first identifying the system as nonsingular.

23. Apply the matrix inverse method to solve a nonsingular linear algebraic system of equations.
SOB: Solve a linear algebraic system of equations using the matrix inverse method by first identifying the system as nonsingular.

24. Comprehend the concepts of a vector space and a subspace.
SOB: Explain that the set of all solutions to a homogeneous linear constant coefficient differential equation is a subspace of the set of all real valued functions differentiable to all orders and defined for all values of the independent variable.

25. Comprehend the concept of a basis for a vector space.
SOB: Explain that the smallest set of solutions to a homogeneous linear constant coefficient differential equation which in linear combination includes all the solutions is a

basis for the vector space defined by all solutions.

26. Comprehend the concept of the dimension of a vector space.
SOB: Explain that all bases for a vector space made up of all solutions to a homogeneous linear constant coefficient differential equation have the same number of solutions called the dimension of the vector space.

The module that challenges the superior students comes from objectives based on the course educational requirements. Three approaches will be incorporated into the module. The first approach takes a method and extends it to a much wider class of problems. In particular, the numerical methods for solving first-order differential equations can be applied to higher order equations. The second approach makes use of an individual project that combines analytical and numerical methods. Thus, several objectives are reinforced. The third approach can be called individual study. Students can choose deeper study of a given topic or explore a related topic and give a report on their findings. These approaches are recommended teaching methods to facilitate learning at the higher cognitive levels (9:18-2). The objectives and samples of behavior for each approach are presented below.

Objectives for Advanced Course Module

Approach 1: Method Extension.

1. Comprehend how an nth-order differential equation can be reduced to a system of first-order differential equations.
SOB: Obtain two first-order differential equations by introducing a reducing variable into a second-order problem.

Test Question: Reduce the given differential equation to an equivalent first-order system.

$$x'' + 2x' + x = 1$$

2. Comprehend how the modified Euler and Runge-Kutta numerical methods approximate a specific solution to a system of first-order differential equations.
SOB: Complete one step of the modified Euler method using the initial conditions to obtain an approximate solution to the system.

Test Question: Use one step of the modified Euler method to approximate x(.1) and y(.1).

$$x' = 2x + y + \exp(t) \qquad x(0) = 1$$

$$y' = x - y + 1 \qquad y(0) = -1$$

Approach 2: Individual Project.

Apply both analytical and numerical methods to study an applied engineering differential equation problem.
SOB: Solve an applied differential equation problem using an analytical and a numerical method.

Test Question: Individual project handout.

Approach 3: Exploration Topics.

Apply an individually studied new method or extension of a method to solve a differential equation or algebra problem.
SOB: Solve a differential equation using an individually studied new method or extension of a method by first discussing why the method is applicable.

Test Question: Individual interview used to discuss the method with posed test problems.

Proposed test questions have been listed with the advanced module objectives. They support the goal of a completely developed module. Test questions have not been itemized in the basic course objectives. The creation of testing materials and detailed lesson materials for the whole course goes beyond the goals set forth in this project. The planning of the basic course to the point where the connection with the module is clearly made, however, will be pursued. This work, together with the development of the module, is presented in Step 4 of the ISD model.

STEP 4: PLAN, DEVELOP, AND VALIDATE INSTRUCTION

The advanced course module must be first planned to fit into the basic course. Then the actual materials for the module will be developed. The topics in the basic course must be organized into a syllabus before accomplishing either task. The study of differential equations logically progresses from first-order to nth-order methods. Advanced topics such as Laplace transforms are then covered. All of the surveyed textbooks used this order. Brauer (3:73,153) integrated the study of numerical methods with the study of each order of differential equation. This approach has been adopted here. The methods are originally introduced to solve first-order differential equations. The linear algebra block

can be considered separately or throughout the course. Both Guterman (4:135-141, 151-161, 175-183) and Rabenstein (6:73-190) introduced linear algebra topics at breaks in the development of differential equations. The portion of linear algebra involving solution methods and matrix techniques will be placed first. This material can be used throughout the course. Also, this block followed by the first-order DE block will give two exams before the academic progress report occurs after Lesson 21. The vector space algebra is placed after the nth-order DE study to reinforce the linear DE results. The syllabus for the basic course, Math 245, with the order already discussed is given in Appendix A. The sections and problems in the syllabus are from Guterman's text (4:--). The book was selected (11:--) based on ease of reading, variety of problems, and the availability of topics for use in a follow-up engineering mathematics course. The time allotted for the topics is based both on the author's previous experience and preliminary discussions with the designated faculty course director (12:--).

The basic course syllabus is next modified to obtain lessons for the advanced course module. The applications lessons, algebra Lesson 6 and nth-order DE Lesson 28, will be deleted. In basic courses, the author has found this type of lesson frequently becomes a catch up or review lesson not needed by the superior student. The first graded review in Lesson 7 has been dropped. The introductory algebra and first-order DE material can be combined into a single exam. The last two graded reviews cover similar blocks of material in both versions of the course. Finally, three pairs of lessons can be combined into three single lessons. The Euler and modified Euler numerical methods, Lessons 13 and 14, now form one lesson. The study of real distinct and real repeated roots in Lessons 21 and 22 can also form a single coherent lesson for the better student. The method of variation of parameters for a higher order differential equation (Lessons 26 and 27) can also be covered in one lesson. These changes provide six lessons for the module.

The three approaches in the advanced course module, described in Step 3 of the ISD model, can now be assigned lessons. Two lessons will be dedicated to the method extension of Approach 1. The first lesson will support the objective related to the reduction technique; the second lesson will extend the modified Euler and Runge-Kutta methods to the reduced system, fulfilling the other objective. One lesson will provide an additional computer laboratory period to support the individual project of Approach 2. This lesson will immediately follow the first two lessons since the individual project builds from the extended numerical methods. The remaining three lessons will

be used for exploratory periods to study topics in Approach 3. These three periods have been placed throughout the second half of the course. The advanced course, Math 245A, Lessons 1 thru 42 are shown in Table 3 with the corresponding lesson number from the basic course whenever possible. The six lesson module is also identified by the approach used. The large overlap of lessons seen in Table 3 illustrates the desired low administrative overhead of the advanced version. The corresponding syllabus for Math 245A is presented in Appendix A.

Lesson Numbers		Lesson Numbers		Lesson Numbers	
Math 245A	Math 245	Math 245A	Math 245	Math 245A	Math 245
1	1	15	18	29	30
2	2	16	19	30	31
3	3	17	20	31	32
4	4	18	21,22	32	33
5	5	19	23	33	34
6	8	20	24	34	35
7	9	21	25	35	36
8	10	22	26,27	36	37
9	11	23	Explore	37	38
10	12	24	Numerical	38	39
11	13,14	25	Numerical	39	Explore
12	15	26	Project Lab	40	40
13	16	27	Explore	41	41
14	17	28	29	42	42

Table 3. Corresponding Lessons From Basic Math 245 Course

The actual materials for the now planned and fitted advanced course module are next developed. The materials for the three approaches previously introduced will be discussed separately.

Numerical techniques for solving a first-order differential equation are extended to one nth-order differential equation in Approach 1. The prepared handout (Appendix A), "Numerical Solution of Nth-order Differential Equations," introduces the concept of reducing one nth-order differential equation to a system of n first-order differential equations. Next, the Euler and modified Euler methods for such a system are presented. In this way the handout provides a logical path to Section 6.7 of the text

where the Runge-Kutta method for a system of differential equations is discussed. Both Approach 1 objectives of comprehending the reduction technique and the numerical techniques applied to a system have then been covered during Lessons 24 and 25.

The individual project of Approach 2 brings both analytical and numerical techniques together in order for the student to study an applied engineering or scientific problem. The project in Appendix A, involving vibrations of a spring and mass system, is just one example of possible problems from many disciplines. In this example, the project introduces the complication of the nonideal spring. Also, the functional definition of the right side forcing function F(t) in Case (b) changes with intervals of time t. This matching difficulty can be removed in the linear case using Laplace transforms studied later. Thus, the project provides an example for later use in the course. The project also makes use of the new network of microcomputers in the cadet dorms. The most interesting results occur if the forcing frequency ω is chosen close to the natural frequency of vibration. For an undamped system, the result will be a beat phenomenon rather than resonance. This phenomenon will occur if the forcing frequency is chosen to equal 0.8 times the natural frequency and the other parameters are selected as follows: m is 0.2 to 0.4 with increment 0.2, k is 4.5 to 5.0 with increment 0.5, F_0 is 1.0 to 1.9 with increment 0.1, and c is 0. These 40 individual projects are ample for an assumed small course size. An extra bonus task could be added by studying the effects of a nonzero damping coefficient c. The supervised laboratory period, Lesson 26, provides time for beginning work on the project.

The third approach requires the individual study of DE or algebra topics selected from a list. The list of some suggested topics with a short summary or description is shown in Appendix A. The topics represent either extensions of material covered in the course or new methods. The topics come from all three blocks in the course. The three periods, Lessons 23, 27, and 39, give the student some consulting or organizing time during class. Each student would select two topics and prepare an informal briefing on each topic for the instructor. The student would provide an overview of the method, would discuss any constraints on the use of the method, and would work an example problem. The instructor could pose other problems to determine whether application level learning has occurred. The available flexibility of choice and time for further study are features in this approach to challenge the student.

The third element of this ISD step is validate the

instruction. Ideally, the materials are presented to a small sample of students in advance of offering the course. This technique appears impractical since the number of available students will already constitute a small percentage of those cadets taking the basic course. The use of individual applied projects by the author in other applied math courses, however, has received very favorable feedback from cadets as a way of stimulating their learning (10:--).

The advanced course module has been designed and the materials have been developed. The goal of this project has been accomplished. The remaining effort would occur in the final ISD step.

STEP 5: CONDUCT AND EVALUATE THE INSTRUCTION

The analysis of requirements, formulation of objectives, and development of materials lead to the actual conduct of the course. The first offering of the advanced version is planned for the fall semester of 1987 and will parallel the first large offering of the basic course. The basic course will be offered for the first time during the current spring semester with less than 100 cadets enrolled. Faculty will evaluate the basic course this spring and incorporate any changes into both versions for the fall of 1987. The advanced course should be kept small with no more than two sections of 14 cadets each. In this way the time will be guaranteed for the significant dialogue between student and instructor.

CONCLUDING REMARKS

The need to provide a more stimulating academic program for the superior cadets at the Air Force Academy has been identified. The concept of inserting a module of lessons into a course to transform it into a more challenging version was developed to meet this need. The modular concept also provides a low administrative overhead cost associated with the advanced version. Three specific approaches for providing more challenge to students were used in the present differential equations course module. Other approaches could also be explored. The modular concept can be applied to a broad range of courses taught at the Academy. In this way the academic programs can be greatly enhanced.

BIBLIOGRAPHY

A. REFERENCES CITED

Books

1. Bloom, B. S. (ed), et al. A Taxonomy of Educational Objectives: Handbook I, The Cognitive Domain. New York: Longmans, Green Company, 1956.

2. Boyce, William E. and DiPrima, Richard C. Elementary Differential Equations and Boundary Value Problems. New York: John Wiley and Sons, 1986.

3. Brauer, Fred and Nohel, John A. Introduction to Differential Equations with Applications. New York: Harper and Row, Publishers, 1986.

4. Guterman, Martin M. and Nitecki, Zbigniew H. Differential Equations: A First Course. New York: Saunders College Publishing, 1984.

5. Powers, David L. Elementary Differential Equations with Boundary Value Problems. Boston: Prindle, Weber, and Schmidt, 1985.

6. Rabenstein, Albert L. Elementary Differential Equations with Linear Algebra. New York: Academic Press, 1982.

Official Documents

7. US Air Force Academy. Spring Semester 1986-1987 Curriculum Handbook. USAFA, CO: HQ USAFA/DF, 1986.

8. US Department of the Air Force. Instructional System Development. AF Manual 50-2. Washington, DC: Government Printing Office, 1986.

9. US Department of the Air Force. Handbook for Air Force Instructors. AF Manual 50-62. Washington, DC: Government Printing Office, 1984.

Unpublished Materials

10. Hegna, Harwood A., Maj, USAF. "Math 355 End-of-Course Report". USAFA/DFMS Letter, 5 June 1986.

11. McLeroy, James C., Maj, USAF. "Text Recommendation for Math 245". USAFA/DFMS Letter, June 1986.

Other Sources

12. Cass, John R., Maj, USAF. Assistant Professor of Mathematical Sciences, Department of Mathematical Sciences, United States Air Force Academy, Colorado. Telecons, October 1986.

13. Lochry, Robert L., Col, USAF, et al. Department Head, Department of Mathematical Sciences, United States Air Force Academy, Colorado. Series of Curriculum Meetings by Department Representatives from Engineering and Basic Sciences Divisions, September-December 1985.

B. RELATED SOURCES

Official Documents

US Department of the Air Force. Handbook for Designers of Instructional Systems. AF Pamphlet 50-58. Washington, DC: Government Printing Office, 1978.

APPENDICES

MATH 245 SYLLABUS SPRING 1987

TEXT: DIFFERENTIAL EQUATIONS: A FIRST COURSE by Martin M. Guterman and Zbigniew H. Nitecki, 1984

LINEAR ALGEBRA SUPPLEMENT (LAS)

VECTOR SPACES CHAPTER 4 SUPPLEMENT (S)

LESSON	SECTION	SUBJECT/HOMEWORK
		BLOCK I
1	1 (LAS)	Introduction to Linear Algebra Work: All problems
2	2, 3 (LAS)	Gauss and Gauss-Jordan Elimination Work: L2 (a1, a3); L3 (a, b1, b4, d)
3	4 (LAS)	Matrix Algebra Work: a, b, c
4	5 (LAS)	Determinants and Cramer's Rule Work: a, b1, b4, c2
5	6 (LAS)	Matrix Inverse Work: a1, b2
6		Applications of Linear Algebra Work:
7		GRADED REVIEW I
		BLOCK II
8	1.1	Introduction to Differential Equations Work: 1, 3, 5, 7, 9, 13, 15, 23
9	1.2	Separation of Variables Work: 1, 3, 5, 13, 15(c), 20
10	1.3	Variation of Parameters Work: 3, 5, 12, 13, 24
11	1.3	Integrating Factor Method Work: 3, 5, 21
12		Applications of First-Order DEs Work:
13	6.2	Euler's Method Work: 1, 3a, 5a

LESSON	SECTION	SUBJECT/HOMEWORK
14	6.4	Modified Euler Method Work: 1, 3a, 5a
15	6.5	Runge-Kutta Method Work: 1, 3a, 5a
16	Handouts	Computer Laboratory
17		GRADED REVIEW II

BLOCK III

LESSON	SECTION	SUBJECT/HOMEWORK
18	2.2	Introduction to n^{th} Order Linear DEs Work: 1, 3, 5, 7, 9, 21
19	2.3	General Properties for Linear DEs Work: 3, 8, 9
20	2.5	Linear Independence of Functions Work: 1, 5, 6
21	2.6	Homogeneous Solutions: Real Distinct Roots Work: 3, 15, 18
22	2.6	Homogeneous Solutions: Real Repeated Roots Work: 5, 9
23	2.7	Homogeneous Solutions: Complex Roots Work: 5, 9, 13
24	2.8	Undetermined Coefficients 1 Work: Page 89 (19, 23); Page 98 (1)
25	2.8	Undetermined Coefficients 2 Work: 4, 5, 13
26	2.9	Variation of Parameters 1 Work: 1, 4, 9
27	2.9	Variation of Parameters 2 Work: 3, 14
28		Applications of n^{th} order DEs Work:
29		GRADED REVIEW III

LESSON	SECTION	SUBJECT/HOMEWORK
		BLOCK IV
30	4.1(S), 4.2(S)	Introduction to Vector Spaces and Subspaces Work:
31	4.3(S)	Spanning Sets Work:
32	4.4(S)	Linear Independence and Bases Work:
33	4.5(S)	Dimension of a Vector Space Work:
34	4.1, 4.2	Introduction to Laplace Transforms Work: 1, 4
35	4.2	Properties of L(x) Work: 9, 11, 12
36	4.2	Inverse Laplace Transforms Work: 15, 17, 19, 20
37	4.3	Initial Value Problems; Partial Fractions Work: 1, 3, 11, 13, 15, 17
38	4.4	Further Transform Properties Work: Page 291 (21); Page 299 (1, 9, 15, 23, 27)
39	4.5	Piecewise Continuous Functions Work: 1, 3, 5, 13, 19, 21
40	4.6	Convolution Integral Work: 1, 3, 12
41		GRADED REVIEW IV
		REVIEW
42		Review/Critique

TEXT: DIFFERENTIAL EQUATIONS: A FIRST COURSE by Martin M. Guterman and Zbigniew H. Nitecki, 1984

LINEAR ALGEBRA SUPPLEMENT (LAS)

VECTOR SPACES CHAPTER 4 SUPPLEMENT (S)

LESSON	SECTION	SUBJECT/HOMEWORK
		BLOCK I
1	1 (LAS)	Introduction to Linear Algebra Work: All problems
2	2, 3 (LAS)	Gauss and Gauss-Jordan Elimination Work: L2 (a1, a3); L3 (a, b1, b4, d)
3	4 (LAS)	Matrix Algebra Work: a, b, c
4	5 (LAS)	Determinants and Cramer's Rule Work: a, b1, b4, c2
5	6 (LAS)	Matrix Inverse Work: a1, b2
6	1.1	Introduction to Differential Equations Work: 1, 3, 5, 7, 9, 13, 15, 23
7	1.2	Separation of Variables Work: 1, 3, 5, 13, 15(c), 20
8	1.3	Variation of Parameters Work: 3, 5, 12, 13, 24
9	1.3	Integrating Factor Method Work: 3, 5, 21
10		Terminology and Solutions - A Pause Work:
11	6.2, 6.4	Euler and Modified Euler Numerical Methods Work: Page 459 (1, 3a, 5a); Page 485 (1, 3a, 5a)
12	6.5	Runge-Kutta Method Work: 1, 3a, 5a
13	Handouts	Computer Laboratory
14		GRADED REVIEW I

LESSON	SECTION	SUBJECT/HOMEWORK
		BLOCK II
15	2.2	Introduction to n^{th} Order Linear DEs Work: 1, 3, 5, 7, 9, 21
16	2.3	General Properties for Linear DEs Work: 3, 8, 9
17	2.5	Linear Independence of Functions Work: 1, 5, 6
18	2.6	Homogeneous Solutions: Real Roots Work: 3, 5, 9, 15, 18
19	2.7	Homogeneous Solutions: Complex Roots Work: 5, 9, 13
20	2.8	Undetermined Coefficients 1 Work: Page 89 (19, 23); Page 98 (1)
21	2.8	Undetermined Coefficients 2 Work: 4, 5, 13
22	2.9	Variation of Parameters Work: 1, 4, 9, 14, 20
23		Exploration Work:
24	Handout	Reduction to First-Order Systems of DEs Work: 1, 2
25	6.7	Numerical Solution of Systems of DEs Work: 1, 3a
26		Computer Laboratory Work:
27		Exploration Work:
28		GRADED REVIEW II

LESSON	SECTION	SUBJECT/HOMEWORK
		BLOCK III
29	4.1(S), 4.2(S)	Introduction to Vector Spaces and Subspaces Work:
30	4.3(S)	Spanning Sets Work:
31	4.4(S)	Linear Independence and Bases Work:
32	4.5(S)	Dimension of a Vector Space Work:
33	4.1, 4.2	Introduction to Laplace Transforms Work: 1, 4
34	4.2	Properties of L(x) and L(Dx) Work: 9, 11, 12
35	4.2	Inverse Laplace Transforms Work: 15, 17, 19, 20
36	4.3	Initial Value Problems; Partial Fractions Work: 1, 3, 11, 13, 15, 17
37	4.4	Further Transform Properties Work: Page 291 (21); Page 299 (1, 9, 15, 23, 27)
38	4.5	Piecewise Continuous Functions Work: 1, 3, 5, 13, 19, 21
39		Exploration Work:
40	4.6	Convolution Integral Work: 1, 3, 12
41		GRADED REVIEW III
		REVIEW
42		Review/Critique

NUMERICAL SOLUTION OF N^{th} ORDER DIFFERENTIAL EQUATIONS

1. The Euler, modified Euler, and Runge-Kutta numerical methods are formulated for solving a first-order differential equation. Suppose we wish to solve an n^{th} order differential equation. If the problem was nonlinear, then none of the analytical methods could be used. Is there a way to use our previously studied numerical methods? The answer is yes provided we can obtain an equivalent problem with first-order differential equations. This method is called reduction of order.

2. Reducing an n^{th} order differential equation to a system of n first-order differential equations is a technique used in many technical disciplines. The method can be illustrated with an example problem.

$$\text{Given} \quad x'' - x = 2t$$

$$x(0) = 1 \quad x'(0) = 2.$$

We can introduce a reducing variable, say y, as follows:

$$y = x'.$$

Then the original second-order problem is reduced one order and becomes

$$y' - x = 2t.$$

The reduced system of 2 first-order differential equations for the 2 unknown functions x(t) and y(t) is

$$x' = y \qquad x(0) = 1$$

$$y' = x + 2t \qquad y(0) = 2.$$

We are now in a position to approximate solutions for 2 curves x(t) and y(t) in an analogous manner to the previously studied numerical methods for one first-order differential equation.

3. SUPPLEMENTAL PROBLEMS:

Find the system of first-order differential equations to each n^{th} order problem.

a. $x''' - 5x'' + 2x' = e^t \qquad x(0) = 1 \qquad x'(0) = 0 \qquad x''(0) = -1$

b. $x'' + 5x' - x^2 = \cos t \qquad x(0) = 2 \qquad x'(0) = 1$

4. The Runge-Kutta method for the numerical approximation of a system is given in Section 6.7 of the text. The key equations for the R-K method are on page 500.

5. Change the program on page 504 for easier use to read:

```
10      DEF FNF (T, X, Y) = Y
15      DEF FNG (T, X, Y) = -9.81 * SIN (X)
20      PRINT "ENTER THE INITIAL VALUE OF 't'"
30      INPUT T
40      PRINT "ENTER THE INITIAL VALUE OF 'x'"
50      INPUT X
55      PRINT "ENTER THE INITIAL VALUE OF 'y'"
56      INPUT Y
60      PRINT "ENTER THE MAXIMUM VALUE OF 't'"
70      INPUT TMAX
80      PRINT "ENTER THE NUMBER OF STEPS"
90      INPUT N
100     H = (TMAX - T)/N
110     A = FNF (T, X, Y)
115     P = FNG (T, X, Y)
120     B = FNF (T + H/2, X + H * A/2, Y + H * P/2)
125     Q = FNG (T + H/2, X + H * A/2, Y + H * P/2)
130     C = FNF (T + H/2, X + H * B/2, Y + H * Q/2)
135     R = FNG (T + H/2, X + H * B/2, Y + H * Q/2)
140     D = FNF (T + H, X + H * C, Y + H * R)
145     S = FNG (T + H, X + H * C, Y + H * R)
150     X = X + (H/6) * (A + 2 * B + 2 * C + D)
155     Y = Y + (H/6) * (P + 2 * Q + 2 * R + S)
160     T = T + H
170     N = N -1
180     If N < = 0 THEN 200
190     GO TO 110
200     PRINT "x("T") = "X
205     PRINT "y("T") = "Y
210     END
```

6. Notice for a system, we run/update/propagate two or more equations at a time. The adjustment of the single DE formulas that will permit handling a system are as follows for Euler and Modified Euler methods.

Euler:

For

$$x'(t) = f(t, x) \quad ; \quad x(t_0) = x_0$$

we used

$$x_i = x_{i-1} + hf(t_{i-1}, x_{i-1}).$$

For a system

$$x'(t) = f_1(t, x, y) \qquad x(t_0) = x_0$$

$$y'(t) = f_2(t, x, y) \qquad y(t_0) = y_0$$

we will use

$$x_i = x_{i-1} + hf(t_{i-1}, x_{i-1}, y_{i-1})$$

$$y_i = y_{i-1} + hf(t_{i-1}, x_{i-1}, y_{i-1}).$$

Modified Euler:

For a system

$$x'(t) = f_1(t, x, y) \qquad x(t_0) = x_0$$

$$y'(t) = f_2(t, x, y) \qquad y(t_0) = y_0$$

we will use

$$x_i^* = x_{i-1} + hf_1(t_{i-1}, x_{i-1}, y_{i-1})$$

$$y_i^* = y_{i-1} + hf_2(t_{i-1}, x_{i-1}, y_{i-1})$$

$$x_i = x_{i-1} + \frac{h}{2}\left[f_1(t_{i-1}, x_{i-1}, y_{i-1}) + f_1(t_i, x_i^*, y_i^*)\right]$$

$$y_i = y_{i-1} + \frac{h}{2}\left[f_2(t_{i-1}, x_{i-1}, y_{i-1}) + f_2(t_i, x_i^*, y_i^*)\right].$$

7. The linear solution in example 6.7.1 on p. 501 is in error. The correct specific solution is $x(t) = \frac{\pi}{4}\cos(\sqrt{9.81}\, t)$.

8. The Summary Table on p. 503 has an error for the value of ℓ_4 which should read $\ell_4 = g(t_{i-1} + h,\ x_{i-1} + hk_3,\ y_{i-1} + h\ell_3)$.

9. SUPPLEMENTAL PROBLEMS:

a. For: $x'' + 8x' + 12x = 3e^{-3t}$ $x(0) = 1,\ x'(0) = 0$

(1) Find the specific solution (by "undetermined coefficients" or "variation of parameters"). Evaluate your exact answer to find x(1).

(2) Reduce to two first-order equations and approx x(1) by the R-K method.

b. Try the R-K process on a system which cannot be solved exactly. You now must be very careful because you have no exact solution for comparison. In practice, most people will claim four-decimal accuracy when increasing the number of time steps by the same proportion (i.e. 2, 4, 8 ...) and repeating the calculation does not change the answer in the fourth decimal.

Problem: $x'' + 2x' - 5x^2 = e^{-t}$ $x(0) = x'(0) = 0$

Find x(1) to 4 decimal accuracy.

DEPARTMENT OF MATHEMATICAL SCIENCES AY
UNITED STATES AIR FORCE ACADEMY
COLORADO SPRINGS, CO 80840

Name____________________ Course Math 245A Section______ Instructor________________

Project Name NUMERICAL PROJECT Date Due________________

ACKNOWLEDGEMENT STATEMENT AND DOCUMENTATION

1. I understand that this is a graded project and that I may:

(a) use my personal class notes, any copyrighted reference, debugging aids provided by the computer, and

(b) consult only with course instructors on the project. On computer projects this restriction includes any aspect of programming, including logic and syntax errors, and

(c) not use this project in any other course without the specific approval of the course director in charge of this course.

2. I further understand that documentation of source materials and documentation of assistance received (including consultation with USAFA instructors) is required in the format prescribed in my MLA Handbook.

a. The source materials I used for this assignment were: (IF NONE, SO STATE)

b. The individuals who provided me with assistance for this assignment were (include type and amount of assistance received): (IF NONE, SO STATE)

c. The existing computer or calculator programs, programming assistance (e.g., modifying existing programs or using existing programs as a guide) and computational equipment used for this assignment were: (IF NONE, SO STATE)

Time Spent on Project Signature Date

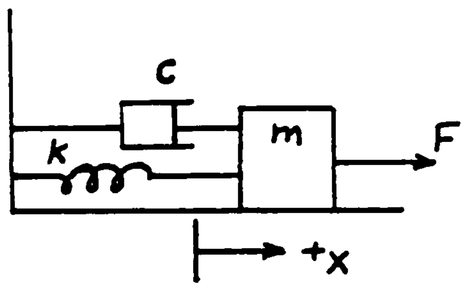
MATH 245A NUMERICAL PROJECT FALL 1987

POINTS: 300 DUE: LESSON 34

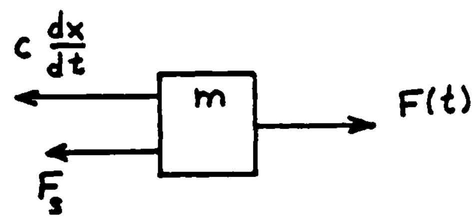
I. Introduction:

1. In many mathematics and engineering problems, we can frequently simplify the problem to a linear form. We then may apply various analytical methods and exactly solve this related problem. Sometimes, however, we require accuracy that ultimately brings us back to the more difficult original problem. In this project we will examine a familiar problem, but we will encounter a real complication. You will solve the problem by using both analytical and numerical techniques.

2. We will look at the damped, spring and mass system with negligible friction shown below.



System



Free Body Diagram

The x coordinate specifies the position of the mass relative to equilibrium at time t. The system is defined by the damping coefficient c, spring constant k, and mass m. An externally applied force F(t) is shown to be positive when in the +x direction. We will consider applied forces that are sinusoidal (Case a) and periodic sawtooth (Case b) defined as follows:

Case a: $F(t) = F_0 \cos \omega t \qquad 0 < t$

Case b:

$$F(t) = \begin{cases} F_0\left(1 - \frac{2\omega}{\pi} t\right) & 0 < t < \frac{\pi}{\omega} \\ F_0\left(\frac{2\omega}{\pi} t - 3\right) & \frac{\pi}{\omega} \le t < \frac{2\pi}{\omega} \end{cases}$$

where F_0 is the amplitude and ω is the angular frequency.

II. Analysis:

1. If we apply Newton's Second Law of Motion to the mass in the +x direction using the free body diagram drawn above, we obtain

$$mx'' = F(t) - cx' - F_s$$

or

$$mx'' + cx' + F_s = F(t). \quad (1)$$

The force exerted by the spring has magnitude F_s and is shown with a direction opposite to the displacement x of the spring. If we assume an ideal spring exists, the spring force varies linearly with the displacement as given by Hooke's Law $F_s = kx$. Equation (1) then becomes

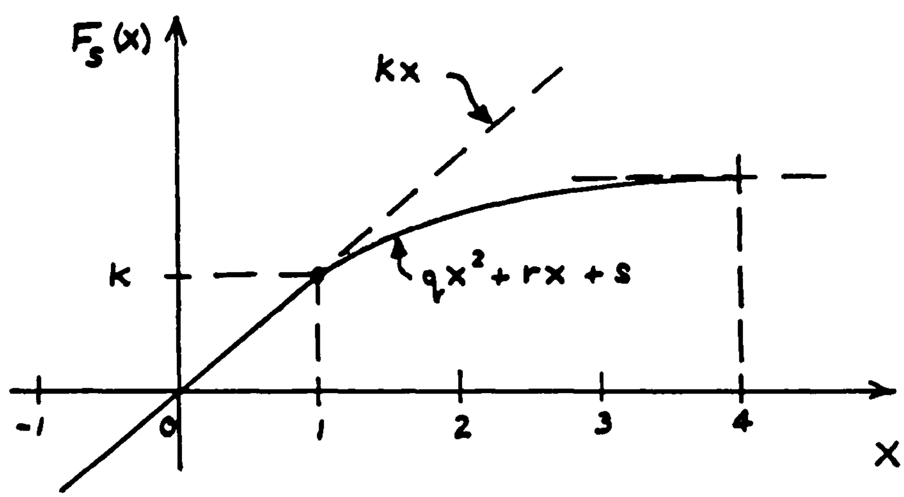
$$mx'' + cx' + kx = F(t) \quad (2)$$

which is the familiar second-order, linear, constant coefficient, nonhomogeneous differential equation of motion for the position x(t) of the mass. We must specify the initial state of the system for either the general case, Eq (1), or ideal spring case, Eq (2). These initial conditions are given by

$$x(0) = x_0 \text{ and } x'(0) = v_0 \quad (3)$$

for initial displacement and velocity, respectively. In this project we will assume an initial equilibrium state $x_0 = 0$ and $v_0 = 0$.

2. Suppose that the spring in our system deviates from the ideal model when extended beyond x = 1. The force is found to increase less with displacement than predicted by Hooke's Law as shown below.



The experimental curve can be represented by a parabola which is tangent to the line and continuous at $x = 1$. Furthermore, the spring reaches yield limit at $x = 4$ where $dF_s/dx = 0$. The spring must not be extended further than $x = 4$. The spring is observed to obey Hooke's Law when compressed $(x < 0)$. The coefficients q, r, and s must be determined from the observed constraints discussed above. The problem has become a second-order, nonlinear, constant coefficient differential equation whenever a spring displacement greater than $x = 1$ occurs! What can cause the spring to extend beyond $x = 1$ since the initial conditions are assumed to be zero?

III. Numerical Formulation:

We can solve nonlinear problems using numerical methods. We must reduce our second-order differential equation, Eq(1), to a system of two first-order differential equations. Let $y = dx/dt$ and obtain the following system:

$$\frac{dx}{dt} = y \qquad x(0) = x_0 = 0$$

$$\frac{dy}{dt} = \frac{1}{m}\left(F(t) - cy - F_s\right) \qquad y(0) = v_0 = 0. \qquad (4)$$

IV. Project Tasks:

Your specific project has the following parameter values:

$m =$ ________, $c =$ ________, $k =$ ________, $F_0 =$ ________, $\omega =$ ________

(10) Task 1: Draw on one graph the applied external forces for Case (a) and Case (b). Indicate their similarities.

(90) Task 2: Use an appropriate analytical method and solve the linear problem given by Eq (2) with zero initial conditions. Consider both Case (a) and Case (b) functions. You may restrict your answer to $0 < t < 2\pi/\omega$.

(10) Task 3: In the force-displacement model for the spring, obtain values for the coefficients q, r, and s by applying the noted constraints. (Hint: The values will contain the spring constant k.) Write the complete description for the function $F_s(x)$.

(150) Task 4: Use the Runge-Kutta method to solve the nonlinear problem for $x(t)$ given in reduced form by Eq(4). Obtain your final approximate solution with $\Delta t = 0.01$ for $0 < t < 4\pi/\omega$. Obtain results for both Case (a) and Case (b) functions.

(40) Task 5: Compare your numerical results for the real nonlinear problem with your analytical results for the ideal linear spring problem. Also compare your results for the similar applied forces in Case (a) and Case (b).

NOTE: You may echo check your results and consult with your course instructor throughout the project.

Objective: Apply an individually studied new method or extension of a method to solve a differential equation or linear algebra problem.

Topics:

1. Variation of Parameters: An alternate form for the particular solution of an initial value differential equation problem involves an integral. The integrand has the right side function and the Green's function for the differential equation. Explore the Green's function method which comes from the variation of parameters linear algebra problem. Look at the properties of a Green's function.

2. Variation of Parameters: In the original derivation, we assumed a particular solution of the form $p(t) = v_1h_1 + v_2h_2$ where h_1 and h_2 are linearly independent homogeneous solutions. Next, we let $v'_1h_1 + v'_2h_2 = 0$ to possibly simplify the problem. Explore setting this expression equal to a known differentiable function $g(t)$ instead and develop a corresponding solution method.

3. Reduction of Order: Reduction of order is a method similar to variation of parameters for solving linear differential equations. The method requires less information than variation of parameters. Explore reduction of order as a solution technique.

4. Method of Exact Differentials: First-order differential equations have a structure similar to the total derivative in multivariable calculus. This similarity may be used to solve a rather complicated looking problem. Explore the method of exact differentials as a solution technique.

5. Matrix Inverse: We studied the reduced echelon approach for computing a matrix inverse when it exists. The adjoint method is another method which frequently appears in analytical work. Explore the adjoint method of finding a matrix inverse.

6. Numerical Method: Many numerical methods exist besides those that we have studied. For instance, here is an approach with elements of both modified Euler and Runge-Kutta. Use the Euler method to obtain an approximation at the midpoint $t_0 + 0.5h$. Next compute the slope at this location. Now use this slope (rather than the average in modified Euler) to go across the interval and obtain a final approximate solution at $t_0 + h$. Explore this numerical method by comparing numerical results with previously studied numerical methods.

LESSONS 23, 27, AND 39: Explorations

1. These lessons are allocated for independent study of the selected topics. Additional out of class time will be necessary. For a given topic, the cadet should prepare an informal briefing to discuss the method, when the method works, and work an example. You might select a new problem for the cadet to solve with the studied method. No written report is required. We will emphasize the ability to apply a self-studied method in the evaluation. Each cadet will work on two topics during the semester.

2. Comments on the topics: Topics 1, 3, and 4 are covered in many differential equations texts. Copies of several texts from our Math Library will be placed on reserve in the Cadet Library. Topic 5 is covered in the majority of linear algebra texts. The numerical method in topic 6 is sometimes called second-order Runge-Kutta or a modified Euler method. This topic, however, can be considered independent of a text. Topic 2 is also an independent analysis subject.

3. We will distribute the topics and complete the selections early in Block 2. No collaboration will be the policy used for the Explorations and will extend throughout the semester since cadets may study the topic at different times. Each cadet must complete one report by Lesson 25 and the second report by Lesson 42.

LESSON 24: Reduction to First-Order Systems of Differential Equations:

Objective: Comprehend how an n^{th} order differential equation can be reduced to a system of first-order differential equations.

1. Our primary goal in Lessons 24 and 25 is to solve a higher order differential equation using our previously studied numerical techniques. Motivate reduction as a means of obtaining first-order differential equations.

2. Work a nonlinear example to illustrate the versatility available.

3. We will not use the matrix system notation since the nonlinear problem does not have this form.

4. You may want to set up a reduced example problem and work through the Euler method to introduce Lesson 25.

LESSON 25: Numerical Solution of Systems of Differential Equations

Objective: Comprehend how the modified Euler and Runge-Kutta numerical methods approximate a specific solution to a system of first-order differential equations.

1. Introduce the extension of the methods to a system with the Euler method. Choose an example problem which can be worked easily with each technique.

2. Emphasize that we are now approximating the graphs of several functions of "t" rather than just x(t).

3. Discuss the relative accuracy of each method. Show the results of one method for several different "Δt"s.

4. The typical problem is a coupled system of differential equations. Show where the numerical methods couple the equations. Discuss how the Euler method only weakly couples the equations compared with Runge-Kutta.

5. Supplemental problems a and b in paragraph 9 of the handout provide a thorough review to this numerical study.

6. The applied differential equation individual project will be handed out at the end of the hour. Cadets should come prepared next lesson to use the microcomputer lab.

LESSON 26: Computer Laboratory

Objective: Apply both analytical and numerical methods to study an applied engineering differential equation problem.

1. The above objective is for the project and not just the lab period. Cadets have the hour for individual computer use.

2. Cadets may echo check all of their numerical results on their project with their instructor. In this way a cadet can obtain learning feedback on the numerical implementation of a method.